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No go theorem for self tuning solutions with Gauss-Bonnet terms

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Abstract: We consider self tuning solutions for a brane embedded in an anti de Sitter spacetime. We include the higher derivative Gauss-Bonnet terms in the action and study singularity free solutions with finite effective Newton's constant. Using the methods of Csaki et al, we prove that such solutions, when exist, always require a fine tuning among the brane parameters. We then present a new method of analysis in which the qualitative features of the solutions can be seen easily without obtaining the solutions explicitly. Also, the origin of the fine tuning is transparent in this method.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions.

Contents

1. Introduction

Randall and Sundrum had proposed a model a few years ago [1] where a $3+1-$dimensional brane is embedded in a five dimensional anti de Sitter (AdS) spacetime. In this model, the brane can be thought of as our universe, with various observable particles assumed to be confined to the brane except graviton which can propagte in the extra fifth dimension also, which is non compact. They showed that for a particular value of the brane tension, with the induced metric on the brane being flat and thus preserving the Poincare invariance on the brane, the zero mode of the graviton is confined to the brane. The effective four dimensional Planck mass M_4 is then finite.

Soon after, the authors of [2] constructed another model where a scalar field ϕ , with a potential $V(\phi)$, can also propagate in the extra dimension. They showed, in this model, that (the zero mode of) the graviton is confined to the brane, preserving the Poincare invariance on the brane, for any value of the brane tension. Taking the brane tenison to be the brane cosmological constant, it therefore follows that a flat Minkowski metric on the brane is possible, preserving Poincare invariance, for any value of the cosmological constant. This, then, could be a solution to the celebrated 'cosmological constant problem' [3].

However, the price one has to pay for this attractive feature is the presence of singularities in the extra dimension at a finite proper distance from the brane. Analysing the solutions explicitly for various choices of $V(\phi)$, it is found that the singularities can be avoided, but then (the zero mode of) the gravity will not be confined on the brane unless the brane tension, equivalently the cosmological constant, is tuned to a specific value. See [4] for other attempts to solve this problem.

In this context, Csaki et al [5] have proved a no go theorem that no singularity free solution with finite M_4, which ensures that gravity is confined on the brane, is possible without a fine tuning. They considered an action for graviton and a scalar field with terms containing atmost two derivatives. Thus, their analysis leaves open a possibility that such solutions may exist for more general action containg higher derivative terms. Also, such terms are expected to appear generically in the effective action upon including the quantum effects of gravity.

Following such a line of reasoning, Low and Zee [6] have considered action with higher derivative terms. They considered a specific combination of such terms, namely the Gauss Bonnet terms, for graviton because of its special features well known in the literature [9]. The other more recent aspects of such terms in the action within the context of brane world phenomenology have been analysed in [8, 7]. The authors [6] analysed various cases explicitly and found in all these cases that no singularity free solution with finite M_4 is possible without a fine tuning.

In this paper, we consider an action containg the higher derivative terms for graviton in the specific Gauss Bonnet combination. The action contains a bulk scalar field ϕ with a potential $V(\phi)$. The brane tension is taken to be an arbitrary function of ϕ. We then study whether a singularity free solution with finite M_4 is possible without a fine tuning.

We prove, following closely the method of [5], that such a solution is not possible. This is the analogue of the no go theorem of [5], but now valid for the case where the action contains the higher derivative Gauss Bonnet terms.

We then present a new method of analysis of the equations involved, reminiscent of the method of 'phase space analysis'. In this method, the qualitative features of the solutions, such as the presence or absence of singularities, finiteness or otherwise of M_4, etc, can be seen easily without obtaining the solutions explicitly. This method is applicable quite generally with or without the higher derivative Gauss Bonnet terms. Moreover, it provides a constructive way of obtaining potentials $V(\phi)$ which will admit singularity free solutions with finite M_4. However, they will all require a fine tuning whose origin is transparent in this method.

The plan of the paper is as follows. In section 2, we set up our notations, present the action, the equations of motion, and the boundary conditions that the solutions must satisfy. In section 3, we establish the no go theorem following the method of [5]. In section 4, we present the method of 'phase space analysis', concluding in section 5 with a brief summary and a few remarks.

2. Preliminaries

We consider $d = 4 + 1$ dimensional spacetime, with coordinates $x^M = (x^\mu, y)$, where $\mu = 0, 1, 2, 3$, $-\infty \leq x^M \leq \infty$, and with a $(3 + 1)$ dimensional brane located at $y = 0$. The bulk fields are the graviton G_{MN} and a scalar field ϕ. Their action, including a higher derivative Gauss-Bonnet term for G_{MN} and a potential V for ϕ, is given by

$$S = \frac{1}{2} \int d^5x \sqrt{G} \Big(R + 4\lambda (R^2 - 4R^{MN} R_{MN} + R^{MNPQ} R_{MNPQ}) - \\ - \frac{3}{4} \left(\partial_M \phi \partial^M \phi + V(\phi) + f(\phi) \delta(y) \right) \Big) , \tag{2.1}$$

where the five dimensional Planck mass is set equal to unity, and the last term denotes the effective tension of the brane located at $y = 0$, with $f(\phi)$ an arbitrary function of ϕ.

We consider warped spacetime solutions, preserving the Poincare invariance along the brane directions x^μ. Thus, the fields are functions of y only. The metric G_{MN} can then be written as

$$ds^2 = e^{-\frac{A(y)}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{2.2}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and $A(y)$ is the warp factor. The curvature invariants are all then functions of $A(y)$ and its derivatives. For example, the Ricci scalar $R = 2W' - \frac{5W^2}{4}$ where we have defined

$$W \equiv A' , \tag{2.3}$$

and the primes, here and in the following, denote differentiation with respect to y. Also, we set $A(0) = 0$ with no loss of generality. $A(y)$ is then given by

$$A(y) = \int_0^y dy W(y) . \tag{2.4}$$

The equations of motion, for $y \neq 0$, that follow from the action (2.1) are

$$W'(1 - \lambda W^2) = \phi'^2 \tag{2.5}$$

$$W^2 (1 - \frac{\lambda W^2}{2}) = \phi'^2 - V \tag{2.6}$$

$$2(\phi'' - W\phi') = V_{(1)} , \tag{2.7}$$

where the subscript (n) denotes the n^{th} differential with respect to ϕ.[1]

The presence of brane at $y = 0$ imposes the following boundary conditions at $y = 0$:

$$\phi_+ = \phi_- \equiv \phi_0 , \qquad \tilde{W}_+ - \tilde{W}_- = 2a , \qquad \phi'_+ - \phi'_- = 2b , \tag{2.8}$$

where the subscripts $\pm$ denote the values at $y = 0_\pm$, $\tilde{W} = (1 - \frac{\lambda W^2}{3})W$, $2a = f(\phi_0)$, and $2b = f_{(1)}(\phi_0)$. The parameters a and b are arbitrary and independent constants since the function $f(\phi)$ is arbitrary. If the values of a and b are restricted, or constrained to obey any specific relation, then they are said to be fine tuned.

Upon substituting the solution for A in the bulk action, and performing the y-integration, one can define an effective four dimensional Planck mass M_4 [6] as follows:

$$M_4^2 \propto \int_{y_L}^{y_R} dy e^{-A(y)/2} , \tag{2.9}$$

where $y_L < 0$ and $y_R > 0$ are the locations of the singularity, if any, closest to the brane on either side. The exact expression which involves a λ dependent factor is given in [7]. If there are no singularities then $y_L = -\infty$ and $y_R = +\infty$. It then follows [1] that if M_4 is finite then (the zero mode of) the graviton is confined to the four dimensional brane at $y = 0$.

[1] Generically, only two of the above equations are indpendent: equations (2.6) and (2.5) imply equation (2.7) if $\phi' \neq 0$; equations (2.6) and (2.7) imply equation (2.5) if $W \neq 0$. Equation (2.6) is, if $\phi' \neq 0$the 'energy' integral of motion for equations (2.5) and (2.7) with the integration constant set to zero.

Our main interest is to study the solutions to equations (2.5)–(2.7), satisfying the boundary conditions (2.8). And, more specifically, to study whether the solutions are free of singularities and have finite M_4. Typically, solutions will have either infinite M_4, or singularities at finite proper distance in the y direction, or both, depending on the potential $V(\phi)$ and the values of the arbitrary constants a and b. Singularity free solutions with finite M_4 are possible, if at all, for only a restricted class of the potentials $V(\phi)$.

In [5], Csaki et al have proved, for actions containing terms with atmost two derivatives, that a singularity free solution with finite M_4, when exists, will always involve a fine tuning. However, effective actions incorporating quantum gravity corrections typically contain terms with higher derivatives. Then, perhaps, it may be possible to obtain singularity free solutions with finite M_4, without any fine tuning.

In this paper, we address this issue. Concretely, we include higher derivative terms for the graviton in the form of Gauss-Bonnet combination. The action is then given by equation (2.1). By an analysis similar to that of [5], we look for solutions with finite M_4 and with no singularities, and study whether any fine tuning is required.

We also present a method of analysis of the equations of motion, reminiscent of the method of 'phase space analysis', where one can determine the presence or absence of singularities, and the finiteness or otherwise of M_4, without having to obtain explicitly the complete solution. This method can also be used to construct $V(\phi)$ which will admit singularity free solution with finite M_4. It turns out that such solution always involves one fine tuning, whose origin can be seen clearly in this method.

3. The no-go theorem

To solve equations (2.5)–(2.7), first consider W as a function of ϕ. Equations (2.5) and (2.6) can then be written as

$$\phi' = (1 - \lambda W^2) W_{(1)} \tag{3.1}$$

$$V = \left(W_{(1)}^2 + \frac{1}{2\lambda} \right) \left(1 - \lambda W^2\right)^2 - \frac{1}{2\lambda} \,. \tag{3.2}$$

In principle, given the potential $V(\phi)$, equation (3.2) can be solved for $W(\phi)$; equation (3.1) can then be solved for $\phi(y)$, and thus for $W(y)$; $A(y)$ is then given by (2.4) [10].

In practice, however, this procedure is of limited use in obtaining general solutions for a given $V(\phi)$ since equation (3.2) is non linear. Instead, one starts with a $W(\phi)$. The corresponding potential $V(\phi)$ is given by equation (3.2). Using equation (3.1), one then obtains $\phi(y)$, and thus $W(y)$ and $A(y)$. Note that the solution thus obtained is not the most general solution to equations (2.5)–(2.7) for the given $V(\phi)$.

We now study, following the method of [5], whether equations (2.5)–(2.7), equivalently (3.1) and (3.2), admit any singularity free solution with finite M_4, for arbitrary values of the constants a and b with no constraints imposed on them - that is, with no fine tuning.

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We first define a conformal coordinate $z(y)$, with $z(0) = 0$, by

$$y = \int_0^z dz \Omega(z) \quad \leftrightarrow \quad dy = \Omega dz \qquad \text{where} \quad \Omega(z) = e^{-\frac{A}{4}} . \tag{3.3}$$

Then, M_4 and the proper distance, $l(y)$, from the brane along the y-direction are given by

$$M_4^2 = \int_{z_L}^{z_R} dz \Omega^3 \qquad \text{and} \quad l(y) = \int_0^y dy = \int_0^z dz \Omega(z) . \tag{3.4}$$

We assume that M_4 is finite and that there is no singularity. Then $0 \le |y| \le \infty$ and, hence, $l(\infty) \to \infty$. Consider the limit $y \to \infty$. (The limit $y \to -\infty$ can be analysed similarly.) Let

$$\Omega \simeq K^{-1} z^q$$

where K is a positive constant. Then, in this limit, it follows that

$$y \simeq \frac{z^{q+1}}{K(q+1)} , \qquad A(z) \simeq -\frac{4q}{q+1} \ln |y| \qquad \text{for} \quad q \neq -1 \tag{3.5}$$

$$y \simeq K^{-1} \ln z , \qquad A(z) \simeq 4K|y| \qquad \text{for} \quad q = -1 . \tag{3.6}$$

The range of q is severely restricted. The requirement that $l(\infty) \to \infty$ and M_4 be finite implies that

$$-1 \le q < -\frac{1}{3} . \tag{3.7}$$

Moreover, $W'(1 - \lambda W^2) \ge 0$ for all values of y (see equation (2.5)). For $q = -1$, this inequality is satisfied since $W' = 0$. For $q \neq -1$, it implies that

$$\frac{4q}{K(q+1)y^2} \left(1 - \frac{16 \lambda q^2}{K^2 (q+1)^2 y^2} \right) \ge 0 .$$

In the limit $|y| \to \infty$ that is being considered here, the second factor is positive and, hence, q must satsify either $q \ge 0$ or $q < -1$. Together, these constraints imply that, in the limit $|y| \to \infty$, $q = -1$ and

$$A(y) \to 4K|y| , \qquad \phi' \to 0 , \qquad \phi \to \phi_c , \tag{3.8}$$

where $\phi_c \equiv \phi_R$ (ϕ_L), for $y \to \infty$ ($-\infty$), is a constant. Using equations (2.3), (3.1), (3.2), and (3.8), it follows that, at $\phi = \phi_c$, $W_{(1)} = 0$ and

$$\mathrm{Sgn}(W) = \sigma , \qquad \mathrm{Sgn} \left(W_{(n)} (1 - \lambda W^2) (\phi')^n \right) = (-\sigma)^{n-1} \tag{3.9}$$

where $\sigma = \mathrm{Sgn}(y)$ and $W_{(n)}$, $n > 1$, is the first nonvanishing derivative of W at ϕ_c. The sign of ϕ', required only when n is odd, is obtained by evaluating ϕ' slightly away from ϕ_c. The relations involving $W_{(1)}$ and W follow directly from equations (3.1) and (3.8). To obtain that involving $W_{(n)}$, we considered a few examples with different, but generic, y-dependences for W which satisfy equation (3.8). $\phi'(y)$ and, thus $\frac{d^n \phi}{dy^n}$, can then be obtained using equation (2.5) and repeated differentiation. Now, equation (3.1) gives

$$\frac{d^n \phi}{dy^n} = W_{(n)} (1 - \lambda W^2) (\phi')^{n-1}$$

where $W_{(n)}$ is the first nonvanishing derivative of W at ϕ_c. Equation (3.9) is obtained, in each of the examples considered, upon comparing the two expressions for $\frac{d^n \phi}{dy^n}$ thus derived.

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We can now evaluate $W_{(n)}(\phi_c)$ for all n, in terms of $V_{(m)}(\phi_c) = \frac{d^m V}{d\phi^m}(\phi_c)$, using equation (3.2). $W(\phi)$ is then given by

$$W(\phi) = \sum_{k=0}^{\infty} \frac{W_{(k)}}{k!}(\phi - \phi_c)^k \tag{3.10}$$

which, by construction, solves the equation (3.2) in the region $0 \le y \le \infty$ for $\phi_c = \phi_R$, and in the region $-\infty \le y \le 0$ for $\phi_c = \phi_L$. (Here and in the following, the coefficients in the Taylor expansions are all to be evaluated at ϕ_c. Their argument ϕ_c will not be written explicitly.) Equations (2.3) and (3.1) can then be used to obtain, in principle, the complete solutions $\phi(y)$ and $A(y)$ for $0 \le |y| \le \infty$. The resulting solution is, by construction, singularity free with finite M_4.

Now, $V_{(n)}$ can be written as

$$V_{(n)} = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} f_{(k)} g_{(n-k)} \tag{3.11}$$

where

$$f_{(0)} = f \equiv W_{(1)}^2 + \frac{1}{2\lambda}\,, \qquad g_{(0)} = g \equiv (1 - \lambda W^2)^2\,.$$

For $n = 0$, 1, and 2, we get, at $\phi = \phi_c$,

$$\begin{aligned} V &= -W^2\left(1 - \frac{\lambda W^2}{2}\right)\,, \qquad V_{(1)} = 0\,, \\ V_{(2)} &= 2W_{(2)}(1 - \lambda W^2)\left(W_{(2)}(1 - \lambda W^2) - W\right) \end{aligned}$$

where we have used $W_{(1)} = 0$. $W_{(2)}$ can be solved in terms of $V_{(2)}$ and admits two branches. However, equations (3.9) rule out one branch and imply, furthermore, that

$$\begin{aligned} V_{(2)}(\phi_c) &\ge 0 \\ 2W_{(2)}(1 - \lambda W^2) &= W - \sqrt{W^2 + 2V_{(2)}} \qquad \text{for} \qquad \phi_c = \phi_R \\ 2W_{(2)}(1 - \lambda W^2) &= W + \sqrt{W^2 + 2V_{(2)}} \qquad \text{for} \qquad \phi_c = \phi_L\,. \end{aligned} \tag{3.12}$$

For $n > 2$, similar expressions can be obtained relating $W_{(n)}$ and $V_{(n)}$. The highest derivative of W that will appear on the right hand side of equation (3.11) is $W_{(n)}$ since $W_{(1)} = 0$. Its coefficient can be easily seen to be given by

$$V_{(n)} = 2W_{(n)}(1 - \lambda W^2)\left(nW_{(2)}(1 - \lambda W^2) - W\right) + \cdots \tag{3.13}$$

where $\cdots$ represent terms involving $W_{(k)}$ with $k < n$, which can all be obtained explicitly by a straightforward combinatorics, but are not necessary for our purposes here.

Note that the coefficient of $W_{(n)}$ never vanishes.[2] This is because n is positive, and $W_{(2)}(1 - \lambda W^2) \le 0$ (≥ 0) when $W > 0$ (< 0), which follows from equation (3.9). Therefore, equation (3.13) can be inverted to obtain $W_{(n)}$ in terms of $V_{(n)}$ and $W_{(k)}$'s, with $k < n$.

[2]Except when $(1 - \lambda W^2) = 0$, a special case which can be easily analysed, with no change in the conclusions.

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These relations can be used iteratively to express $W_{(n)}$ in terms of $V_{(k)}$, $k \leq n$, for all n. Equation (3.10) then gives the functions $W_R(\phi)$ and $W_L(\phi)$ which, by construction, solves the equation (3.2) in the region $0 \leq y \leq \infty$ for $\phi_c = \phi_R$, and in the region $-\infty \leq y \leq 0$ for $\phi_c = \phi_L$ respectively. Thus

$$\begin{aligned} W_R(\phi) &= \sum_{k=0}^{\infty} \frac{W_{(k)}(\phi_R)}{k!}(\phi - \phi_R)^k \\ W_L(\phi) &= \sum_{k=0}^{\infty} \frac{W_{(k)}(\phi_L)}{k!}(\phi - \phi_L)^k \,. \end{aligned} \tag{3.14}$$

The boundary conditions (2.8) are yet to be imposed where the parameters a and b are arbitrary constants. The equation involving a implies that

$$\left(1 - \frac{\lambda W_R^2(\phi_0)}{3}\right) W_R(\phi_0) - \left(1 - \frac{\lambda W_L^2(\phi_0)}{3}\right) W_L(\phi_0) = 2a$$

where $W_R(\phi)$ and $W_L(\phi)$ are given by equations (3.14). This condition fixes the value of ϕ_0 in terms of the parameter a. Once ϕ_0 is fixed, the discontinuity in ϕ', namely,

$$\phi'_+ - \phi'_- = \left(1 - \lambda W_R^2(\phi_0)\right) W_{R(1)}(\phi_0) - \left(1 - \lambda W_L^2(\phi_0)\right) W_{L(1)}(\phi_0)$$

is also fixed. This, in turn, implies that the parameter b can not be arbitrary, but must be related to the parameter a as given by the above set of equations. Hence, it follows that singularity free solution(s) with finite M_4 will involve a fine tuning.

Note that ϕ_R and ϕ_L are not arbitrary but must be the values of ϕ for which $V(\phi)$ is a minimum. (See equation (3.12).) Thus, they can take only a discrete set of values and, in particular, cannot be continuos parameters of the solution. The boundary conditions above may also allow for a discrete set of values for ϕ_0. Hence, a similar discrete range of values is also allowed for the parameter b. Nevertheless, the values of a and b must be related as given by the above set of equations and, hence, there must be a fine tuning. Various subtleties that may arise regarding the choice of the branch(es) of $W(\phi)$, etc. are analysed in detail in [5]. The same analysis remains valid in the present case also, where the higher derivative Gauss-Bonnet term is included in the action. Thus, we conclude that singularity free solutions with finite M_4 requires a fine tuning. This is the analogue of the no go theorem of [5], but now valid for the case when the action contains higher derivative Gauss Bonnet terms.

4. Another method of analysis

We now present another method of analysis of the equations involved, reminiscent of the method of 'phase space analysis'. In this method, the qualitative features of the solutions, such as the presence or absence of singularities, finiteness or otherwise of M_4, etc, can be seen easily without obtaining the solutions explicitly. This method also provides a constructive way of obtaining potentials $V(\phi)$ which will admit singularity free solutions with finite M_4. However, they will all require a fine tuning whose origin is transparent in this method.

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Consider W as a function of ϕ. Then one can, in principle, obtain $\phi(W)$ and thus $W_{(1)}(W)$ as functions of W.[3] $V(W)$ is then given by (3.2). It follows, from equations (2.5) and (2.6), that

$$W' = \frac{V + W^2(1 - \frac{\lambda W^2}{2})}{1 - \lambda W^2} \qquad \text{with} \qquad V + W^2 \left(1 - \frac{\lambda W^2}{2}\right) \geq 0\,, \tag{4.1}$$

from which $W(y)$ can, in principle, be obtained since V is now a function of W. $\phi(y)$ follows since ϕ too is a function of W.

Using $dy = dW\left(\frac{1}{W'}\right)$, with $W'(W)$ given by (4.1), we have

$$y = \int dW \left(\frac{1}{W'}\right), \qquad A = \int dW \left(\frac{W}{W'}\right), \qquad M_4^2 = \int dW \left(\frac{e^{-\frac{A}{2}}}{W'}\right)$$
$$\phi = \int dW \frac{1}{W'} \left(V + W^2 \left(1 - \frac{\lambda W^2}{2}\right)\right)^{\frac{1}{2}}.$$

Thus, the required quantities are all written as integrals of functions of W with respect to W. Then, various qualitative features, such as presence or absence of singularities, finiteness or otherwise of M_4 can all be obtained, essentially, by inspection and simple asymptotice analyses in suitable limits.

We illustrate this approach by an example. Let $V = -\alpha^2$, with $0 < 2\lambda\alpha^2 < 1$. The plot of $\frac{dy}{dW} = \frac{1}{W'}$ vs W is given in figure 1 where only the regions satisfying $W^2(1 - \frac{\lambda W^2}{2}) \geq \alpha^2$ are allowed. That is, the allowed regions are given by

$$|a| \leq |W| \leq |b|$$

where a and b, both taken to be positive, are given by

$$a^2 = \frac{1 - \sqrt{1 - 2\lambda\alpha^2}}{\lambda}\,, \qquad b^2 = \frac{1 + \sqrt{1 - 2\lambda\alpha^2}}{\lambda}\,.$$

The plot of y vs W can then be obtained from figure 1 by inspection and a simple asymptotic analysis in the limits $|\frac{dy}{dW}| \to \infty$ and/or $|W| \to \infty$. It is shown in figure 2, upto a constant shift of different branches in the y-direction.

Let $W = W_+$ at $y = 0_+$. Clearly, W_+ must lie in the allowed region. Consider the evolution for $y > 0$. (A similar analysis can be done for $y < 0$ with $W = W_-$ at $y = 0_-$.) Since y must increase, the direction of evolution must be along the branch which contains W_+, and must be in the upward direction. It is clear, from figure 2, that either

(i). $y \to \infty$ and W tends to a negative value, or

(ii). y tends to a finite value and $\frac{dy}{dW} \to 0$, or

(iii). y tends to a finite value and $W \to \infty$. W itself may increase or decrease depending on its initial value W_+.

[3] For example, if $W = \alpha + \beta\phi^n$ then $W_{(1)} = n\beta \left(\frac{W-\alpha}{\beta}\right)^{\frac{n-1}{n}}$.

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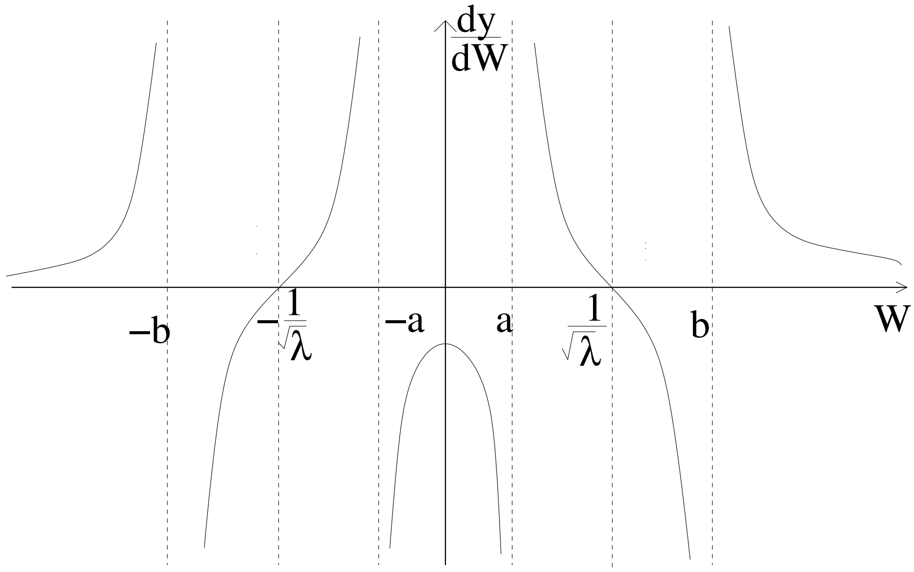

Figure 1: Plot of dy/dW vs W, for $V = -\alpha^2$. Allowed regions are $|a| \le |W| \le |b|$.

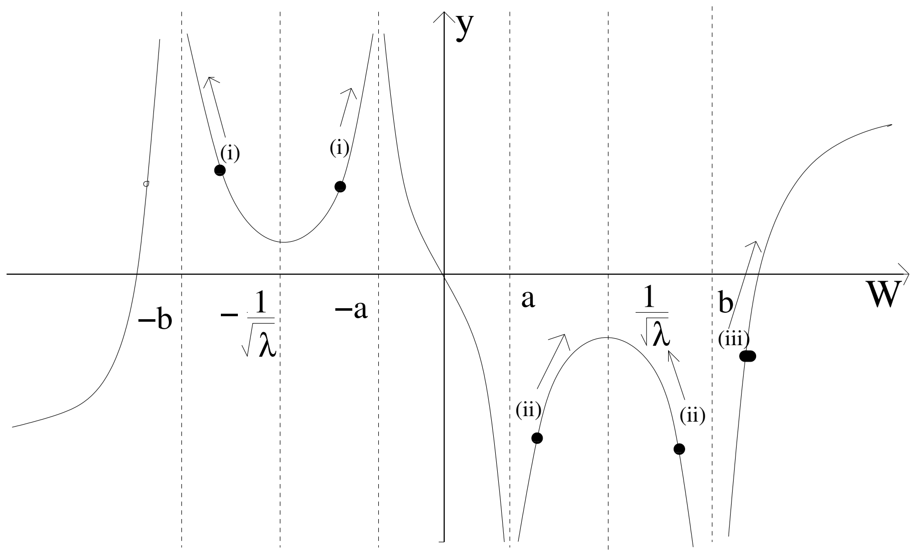

Figure 2: Plot of y vs W, obtained qualitatively from figure 1. Allowed regions are $|a| \le |W| \le |b|$.

In case (i), $M_4 \to \infty$ find W tends to a negative value. See eq (9). In case (ii), there is a singularity since W' and, hence, the Ricci scalar R diverge. In case (iii) also,[4] there is a singularity since W and, hence, R diverge. Thus, for $V = -\alpha^2$, it is immediately clear

[4]This case will not arise in the present example since the corresponding branch lies outside the allowed region. Generically, however, it will.

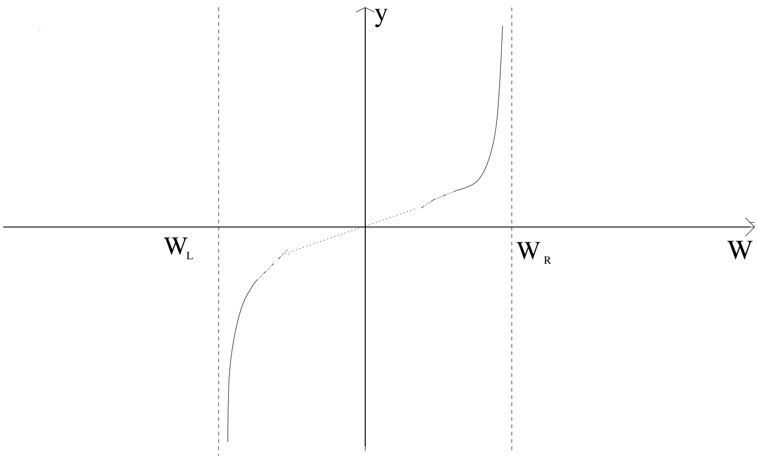

Figure 3: An example of a function $y(W)$ with desireable properties.

that, irrespective of the values of a and b in (2.8) which only determine $W_{\pm}$, the solutions will either have a divergent M_4, or a singularity at a finite distance from the brane, or both.

Similar analysis can be performed in all cases where one starts with a $W(\phi)$. Note that no explicit solutions $\phi(y)$ and $A(y)$ are needed to determine the presence or absence of singularities, and the finiteness or otherwise of M_4. Perhaps, the only difficult step is in inverting the given function $W(\phi)$ to obtain $\phi(W)$, and thus $W_{(1)}(W)$ and $V(W)$, as functions of W.

Using this method of analysis, we can easily construct a whole class of potentials $V(\phi)$ for which there are solutions with finite M_4 and no singularities. One first determines a suitable function $y(W)$ with the desireable properties: Clearly, in the $y - W$ plane, there must be one branch where $y \to \infty$ and $W \to W_R > 0$ with $0 < W_R < \infty$; and another branch where $y \to -\infty$ and $W \to W_L < 0$ with $-\infty < W_L < 0$, neither of the branches containing any critical point where $\frac{dy}{dW} \to 0$. See figure 3. This will then ensure that for some continuous, non trivial, ranges of the parameters a and b in (2.8), there exist singularity free solutions with finite M_4.

Once a suitable function $y(W)$ is chosen, equations (4.1) and (3.2) will give $V(W)$ and $W_{(1)}(W)$. $V(\phi)$ can then be obtained by a series of straightforward operations such as differentiation, integration, functional inversion, etc.

We now illustrate such a construction by an example. A simple function $y(W)$, with all the properties mentioned above, is the one in figure 3 where the two branches are connected with no critical point in between, and $W_R = -W_L = \beta$ with $0 < \beta < \infty$. A simple choice for $y(W)$ is given, for example, by

$$\frac{dy}{dW} = \frac{1}{\alpha^2(\beta^2 - W^2)}, \tag{4.2}$$

with the allowed region given by $(\beta^2 - W^2)(1 - \lambda W^2) \geq 0$.

Let $\lambda = 0$ and $\alpha = \beta = 1$ in equation (4.2). It then follows that $V = 1 - 2W^2$ and $W_{(1)}^2 = 1 - W^2$. Solving this, one obtains $W = \epsilon sin\phi$ and $V(\phi) = cos2\phi$, where $\epsilon = \pm 1$. The explicit solutions for $\phi(y)$ and $W(y)$ is given by

$$\begin{aligned} W &= \epsilon_{\pm} \sin\phi = \tanh(y + y_{\pm}) \\ \phi' &= \delta_{\pm} \operatorname{sech}(y + y_{\pm}) \end{aligned} \tag{4.3}$$

where $\pm$ signs indicate that the solutions are valid for positive or negative values of y, and the ϵ's and δ's take values ± 1. It is easy to see that these solutions are singularity free with finite M_4. Upon imposing the boundary conditions (2.8), $\epsilon_{\pm}$ and $\delta_{\pm}$ can be determined, and $y_{\pm}$ can be obtained in terms of a and b. However, it turns out that either $ab = 0$ (that is, either a, or b, or both vanish) or $a^2 + b^2 = 1$. Any of these conditions amounts to a fine tuning. If this fine tuning relation is not satisfied then, either M_4 will diverge, or there will be singularities, or both. To show this requires a further analysis which, however, is beyond the scope of the present work and will be described elsewhere.

Furthermore, solutions in (4.3) also obey the equation $W^2 + \phi'^2 = 1$ everywhere, which is not part of the equations of motion, but is precisely the one obtained from $V = 1 - 2W^2$. Note also that the values of ϕ at $y = \pm\infty$ is given by $\phi_c = n\pi + \frac{\pi}{2}$ and that $V_{(2)}(\phi_c) = -cos(2\phi_c) > 0$, as required by the general analysis in section 3.

Let $\lambda > 0$, $\alpha^2 = \lambda s^2$, and $\beta^2 = \lambda^{-1}$ in equation (4.2). It then follows that $W = s\phi$, which is precisely the case analysed in [6] where it is shown that there exist singularity free solutions with finite M_4. However, the constants a and b are not arbitrary, but must be fine tuned. Furthermore, these solutions also obey the equation $\phi' = s(1 - \lambda W^2)$, which is not part of the equations of motion but is precisely the one obtained from equation (3.1) using $W = s\phi$.

Thus, we have a method of constructing the potentials $V(\phi)$, which admit singularity free solutions with finite M_4 for some continuous, non trivial, ranges of the parameters a and b in (2.8). However, there will always be an extra relation imposed on the arbitrary constants a and b. This, indeed, is fine tuning.

The origin of this extra condition and, thus, of the fine tuning is clear in the present approach. It arises because V and, hence, $W_{(1)}$ are specific functions, which depend on the choice of $y(W)$, which in turn was constructed to ensure that there exist singularity free solutions with finite M_4 for some continuous, non trivial, ranges of the parameters a and b. Since $W_{(1)}$ is related to ϕ', as given by (3.1), $W_{(1)} = W_{(1)}(W)$ implies a new equation for ϕ' and $A'(= W)$, which is not part of the equations of motion (2.5)–(2.7) obtained from the action.[5] namely $\phi' = (1 - \lambda A'^2) W_1(A')$. This is the origin of fine tuning.

5. Conclusion

In this paper, we considered an action containing the higher derivative terms for graviton in the specific Gauss Bonnet combination and studied whether a singularity free solution with finite M_4 is possible without a fine tuning. We proved that such a solution is not

[5]As noted in the above examples, one gets $\phi'^2 + W^2 = 1$ in the $\lambda = 0$ case, and $\phi' = s(1 - \lambda W^2)$ in the $\lambda > 0$ case.

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possible. This is the analogue of the no go theorem of [5], but now valid for the case where the action contains the higher derivative Gauss Bonnet terms.

We provided a new method of analysis of the equations involved in which the qualitative features of the solutions, such as the presence or absence of singularities, finiteness or otherwise of M_4, etc, can be seen easily without obtaining the solutions explicitly. This method is applicable quite generally and provides a constructive way of obtaining potentials $V(\phi)$ which will admit singularity free solutions with finite M_4. However, such solutions will all require a fine tuning, consistent with the present no go theorem and that of [5]. The origin of the fine tuning is transparent in this method.

This no go theorem is likely to be valid for all higher derivative terms. However, it is not clear how to extend the proof for the most general case. One hurdle, among possibly many others, is that the equations of motion will involve derivatives higher than two. (The case of Gauss Bonnet terms is an exception. See [6].) Also, such terms will invovle ghosts upon quantisation. Their physical implication is then not clear. Nevertheless, considering the importance of the issue, it is desireable to establish, if true, the present no go theorem for the most general case. Its failure, if happens, will also be very interesting as it may provide further insights into the cosmological constant problem.

The equations of motion analysed here appear in other contexts also with minor modifications. For example, they appear in the renormalisation group flow of field theories in the AdS/CFT correspondence [11] and in the cosmological evolution of a $(3+1)-$dimensional universe [12]. It will be of interest to explore these connections and, in particular, to study the implications of the present no go theorem in the above mentioned contexts.

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